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Chiral effective potential in $\mathcal{N} = 1/2$ non-commutative Wess-Zumino model

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Abstract: We study a structure of holomorphic quantum contributions to the effective action for $\mathcal{N} = 1/2$ noncommutative Wess-Zumino model. Using the symbol operator techniques we present the one-loop chiral effective potential in a form of integral over proper time of the appropriate heat kernel. We prove that this kernel can be exactly found. As a result we obtain the exact integral representation of the one-loop effective potential. Also we study the expansion of the effective potential in a series in powers of the chiral superfield Φ and derivative $D^2\Phi$ and construct a procedure for systematic calculation of the coefficients in the series. We show that all terms in the series without derivatives can be summed up in an explicit form.

Keywords: Superspaces, Non-Commutative Geometry.

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1. Introduction

The deformation of superspace and construction the Moyal superstar product based on non-trivial (super)Poisson manifolds [1], as a first step in constructing supersymmetric string field theory, has been attracted much attention. It has been shown [2] that in the language of the string field theory the Moyal product is the simplest description of bosonic string interactions, corresponding to strings joining or splitting. In the recent literature there are other fundamental studies of a star product in a certain class of quantum field theories on noncommutative (NC) Minkowski space-times, which origin is the Seiberg-Witten limit of open strings in the presence of an external constant NS-NS B-field [3]. To be more precise, by wrapping the branes with non-zero constant background field B_{mn} we get the corresponding low energy effective gauge theory which is deformed to a noncommutative (super)symmetric gauge theory in such a way that (bosonic) directions become noncommutative. This result generated a modern activity in studying quantum field theories in NC space (see [4, 5, 6] for reviews). Also we point out that noncommutative field theories provide a simplified ground for investigating the nonlocal string theory effects.

Interplay between noncommutative field theories (NCFT) and string theories has been a rich and fruitful source for better understanding of both. Part of the interest has been aimed at getting the new insights into the regularization and renormalization of quantum

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field theories in this novel framework that are neither local nor Lorenz invariant. Another significant interest is conditioned by the relation between classical and quantum nonlocality in these theories. In fact, many features of ordinary (commutative) field theories have found rich analogues in the NC context. The first revealed unexpected property of NCFT [7] is that even in massive theories there is (UV/IR) mixing of ultraviolet and infrared divergences. As a consequence, the Wilsonian approach to field theory seems to break down: integrating out hight-energy degrees of freedom produces unexpected low-energy divergences.

For the reasons mentioned above the elementary quanta in NCFT are no longer point particles; instead of it, the physical excitations are described by the NC-dipoles — weakly interacting, one dimensional extended objects [8]. In a generic NCFT always exists a special class of composite operators: open Wilson lines $W_k(\Phi)$ and their descendants $(\Phi W)_k(\Phi)$. Infrared dynamics of NC dipoles and hence the open lines is dual to ultraviolet dynamics of the elementary fields Φ's.

Furthermore in NC gauge theories (NCGT), which appear naturally in various decoupling limits of the worldvolume theories of D-branes in background NS-NS B-field, the gauge invariance becomes subtle. The translations along NC directions are a subset of gauge transformations and thus there are no gauge invariant local operators in the position space. It turns out that NC gauge theories allow a new type of gauge invariant objects which are localized in the momentum space (see for reviews [4, 8, 9]).

The open Wilson line is defined in terms of a path-ordered $\star$-product, and its expansion in powers of the gauge potential involves a generalized $\star_n$-product at each n-th order. Generalization of the $\star$-product appears in the one-loop effective action of NCGT [10, 11], in couplings between the massless closed and open string modes [12], and in Seiberg-Witten map between the ordinary and NC Yang-Mills fields [13]. So it has been found a complete agreement of the results with the Seiberg-Witten limit of the string world-sheet computation and standard Feynman diagrammatic at low-energy and the large noncommutative limit [10, 11].

More recently, the extension of noncommutativity to odd variables has been related to the presence of other background fields. In particular, in [14] it has been studied the C-deformation of $\mathcal{N}=1$ supersymmetric gauge theories in four dimensions and computed the coupling to the graviphoton superfield $F^{\alpha\beta}$ arising from the higher-genus amplitudes in the (topological) superstring theory. They introduced the noncommutativity only in the Grassmann odd coordinates, which breaks spacetime supersymmetry explicitly. New type of a deformation has been proposed by Seiberg [15] who introduced noncommutativity both in Grassmann even and odd coordinates but imposed the commutativity in the chiral coordinates. This deformation is made by such a way that the anticommuting coordinates θ form a Clifford algebra

$$\{\hat{\theta}^\alpha, \hat{\theta}^\beta\} = 2\alpha'^2 F^{\alpha\beta} = C^{\alpha\beta}\,. \tag{1.1}$$

The other commutation relations are determined by the consistency of the algebra. In particular, the ordinary spacetime coordinates x^m can not commute

$$[\hat{x}^m, \hat{\theta}^\alpha] = iC^{\alpha\beta}\sigma^m_{\beta\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\,, \qquad [\hat{x}^m, \hat{x}^n] = \bar{\theta}\bar{\theta}C^{mn}\,, \qquad \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = 0\,, \tag{1.2}$$

where $C^{mn} = C^{\alpha\beta}\varepsilon_{\beta\gamma}\sigma^{mn\,\gamma}_{\alpha}$. In contrast to the spacetime coordinates, the chiral coordinates $y^m = x^m + i\theta^\alpha\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}$ can be taken commuting. Note that because the anticommutation relation of $\bar\theta$ remains undeformed, $\bar\theta$ is not the complex conjugate of θ, that is possible only in the euclidean space. The product of functions of θ on the chiral superspace is Weyl ordered by using the star-product, which is the fermionic version of the Moyal product:

$$f(\theta)\star g(\theta) = f(\theta)\exp\left(-\frac{1}{2}\,C^{\alpha\beta}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta}\right)g(\theta)\,. \tag{1.3}$$

The supercharges are defined as follows

$$Q_\alpha = i\frac{\partial}{\partial\theta^\alpha}\,, \qquad \bar Q_{\dot\alpha} = i\left(\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\alpha\frac{\partial}{\partial y^{\alpha\dot\alpha}}\right). \tag{1.4}$$

We use the conventions of [16] and therefore we expect Q_α have to be symmetry generators on the $\mathcal{N} = 1/2$ supersymmetry. The star-product (1.3) is invariant under the action of supercharges Q_α. However, because $\bar Q_{\dot\alpha}$ depends explicitly on θ, it is clear that the star-product is not invariant under $\bar Q$. Such a deformation saves the $\mathcal{N} = 1/2$ supersymmetry and has interesting properties in the field theory viewpoint. Replacing all ordinary products with the above $\star$-product, one can proceed studying a supersymmetric field theory in this non(anti)commuting superspace taking into account that this deformed supersymmetry algebra admits well-defined representations. Namely, one can define chiral and vector superfields much similarly to the ordinary $\mathcal{N} = 1$ supersymmetry [15].

It is very interesting to study how the deformation (1.1) modifies the quantum dynamics of supersymmetric field theories, paying particular attention to consequences of nonlocality in the superspace caused by eq. (1.3). It was pointed that the deformation (1.3) induces local operators multiplied by the non(anti)commutativity parameter $C^{\alpha\beta}$ [15]. However, even though C breaks Lorentz invariance, the theory with an arbitrary superpotential $W(\Phi)_\star$ is Lorentz invariant because the deformation depends only on $\det(C)$. These induced operators have higher scaling dimension (dim = 6), and they might cause the theories to be nonrenormalizable.

Some recent papers deal with various aspects of field theories defined on such NC superspace [15], [17]–[20]. More recently, also the instanton configurations of the $\mathcal{N} = 1/2$ gauge theory have been analyzed [21]. Theories with nilpotent deformation of $\mathcal{N} = 2$ supersymmetry have been constructed in [22].

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Though new kinds of (anti)chiral superfields in $\mathcal{N} = \frac{1}{2}$ supersymmetric theory violate the holomorphy, the anti-holomorphy still remains. For deformed WZ-model, this leads

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to the non-renormalization theorem of the anti-chiral superpotential and vanishing of the vacuum energy. Moreover, one can show that such deformed theories have finite number of divergent structures in their effective actions and hence, they are in fact renormalizable. Besides, unlike the $\mathcal{N} = 1$ supersymmetric models, in $\mathcal{N} = 1/2$ models containing the chiral and antichiral superfields exist the loop corrections to the chiral effective action even in a massive case (see for comparison a situation in standard $\mathcal{N} = 1$ SUSY models [23] and in WZ model with noncommutative spacetime [24]. Theories with nilpotent deformation of $\mathcal{N} = 2$ supersymmetry have been constructed in [22].

One more important observation was that the deformed WZ-model after adding the new F and F^2 terms (which are not written in the star deformation) to the original lagrangian becomes renormalizable up to two loops [17]. The renormalizability was then extended to all orders in the perturbation theory for the deformed Wess-Zumino model in [18, 19], and for deformed gauge theories with(out) matter in [20]. In particular, these works show that, thought these terms carry scaling dimensions large than four, the deformation-induced operators do not lead to power divergences in loop diagrams because of absence the hermitean conjugate operators to them and they are radiatively corrected at least logarithmically. Also it was shown that all divergent terms have one power insertion of $\det(C)$ and counterterms F, F^2, F^3 suffice to renormalized the theory. Using nonstandard scaling dimension assignment, in ref. [25] has been explain intuitively renormalizability of the NCFT. With insertions of all possible undeformed and deformed terms having the dimensions less then or equal to four in the lagrangian, the theory under consideration is multiplicatively renormalizable.

Other remarkable quantum properties of supersymmetric theories such as stability of the vacuum energy and the existence of an antichiral ring, remain unchanged despite of the fact that NC induced a soft-breaking of supersymmetry. The vacuum energy remains zero to all orders in perturbation theory. Structure of the effective action in the $\mathcal{N} = 1/2$ WZ model has been studied in [26] where an analysis for summing the one-loop contributions to the effective potential has been presented for the case of small odd and even external momenta. Unfortunately, it is unclear how to interpret the physical sense of such approximations.

In this work we develop a general approach to constructing the one-loop effective potential in $\mathcal{N} = 1/2$ WZ model. The approach is based on use of the symbol operator techniques and heat kernel method and allows to carry out a straightforward calculation of one-loop finite quantum corrections. As a result we find an exact form of one-loop effective potential for the considered model in terms of a proper-time integral. Also we construct a new scheme for approximate evaluation of the effective potential and give a complete solution of the problem settled up in [26].

The paper is organized as follows. Section 2 is devoted to a formulation of the model. In section 3 we describe a general procedure of calculations. Subsection 3.1 is devoted to a brief discussion of the symbol operator techniques and its application to finding the one-loop effective action for the theories in $\mathcal{N} = 1$ superspace. In subsection 3.2 we calculate the exact heat kernel and find the one-loop effective potential for the theory under consideration. The effective potential provides the complete solution of the problem

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formulated in [26]. Subsection 3.2 is devoted to constructing a scheme for approximate calculation of the effective potential in a form of an expansion in the field Φ and derivatives $D^2\Phi$. In section 4 we present a procedure for an explicit calculation of the terms in the above expansion. Section 4.1 is devoted to finding the divergences in the framework of our general scheme. Explicit calculations of two first finite terms in the expansion of the effective potential are presented in the subsection 4.2.1 and subsection 4.2.2. It is interesting to point out that these terms as well as the others can be expressed in terms of hypergeometric functions. In subsection 4.3 we construct an approximation form of the effective potential by summing up all terms including all powers of superfield Φ but do not containing the derivatives $D^2\Phi$. Summary is devoted to discussion of the results obtained.

2. The model

On the $\mathcal{N}=1/2$ noncommutative superspace the WZ model is given by the standard classical action where the point products of superfields are replaced with the star product (1.3):

$$S=\int d^8z\,\bar{\Phi}\star\Phi+\int d^6z\left(\frac{m}{2}\Phi\star\Phi+\frac{g}{3!}\Phi\star\Phi\star\Phi\right)+\int d^6\,\bar{z}\left(\frac{\bar{m}}{2}\bar{\Phi}\star\bar{\Phi}+\frac{\bar{g}}{3!}\bar{\Phi}\star\bar{\Phi}\star\bar{\Phi}\right). \tag{2.1}$$

Chiral superfields are defined by the relation $\bar{D}_{\dot{\alpha}}\Phi(y,\theta,\bar{\theta})=0$, which means $\Phi(y,\theta)=A(y)+\theta\kappa(y)+\theta^2F(y)$. Anti-chiral superfields are defined by the relation $D_\alpha\bar{\Phi}(y,\theta,\bar{\theta})=0$, which means $\bar{\Phi}(\bar{y},\bar{\theta})=\bar{A}(\bar{y})+\bar{\theta}\bar{\kappa}(\bar{y})+\bar{\theta}^2\bar{F}(\bar{y})$. As it has been demonstrated in ref. [15], the $\star$-product of the chiral superfields is again a chiral superfield; likewise, the $\star$-product of the antichiral superfields is again an antichiral superfield.

The model is formulated in euclidean space where the fields $\Phi,\bar{\Phi}$ are considered as independent. Using the property $\int\Phi\star\Phi=\int\Phi\cdot\Phi$, performing the expansion of the star-product (1.3) and turning down total superspace derivatives, the cubic interaction terms reduce to the usual WZ interactions complemented by the terms violating $\mathcal{N}=1$ supersymmetry to $\mathcal{N}=1/2$ supersymmetry.

$$\begin{aligned}S=&\int d^8z\,\bar{\Phi}\Phi+\int d^6z\left(\frac{m}{2}\Phi\Phi+\frac{g}{3!}\Phi\Phi\Phi\right)+\int d^6\bar{z}\left(\frac{\bar{m}}{2}\bar{\Phi}\bar{\Phi}+\frac{\bar{g}}{3!}\bar{\Phi}\bar{\Phi}\bar{\Phi}\right)+\\&+\int d^6z\left(\frac{h}{3!}\Phi(Q^2\Phi)^2+\frac{1}{2\lambda}\Phi(Q^2\Phi)\right),\end{aligned} \tag{2.2}$$

where $h=-\frac{g}{4}|\det C|$. Last term containing the coupling λ is added to provide a multiplicative renormalization of the model (see e.g [18]). As a result we see that the action (2.1) is rewritten in terms of standard $\mathcal{N}=1$ superspace, i.e. without star-product. Hence, one can consider the deformed WZ model as ordinary WZ model, where superfield multiplication is standard, with a new addition to the F-term.

Thus we treat the theory as some special model formulated in terms of $\mathcal{N}=1$ superspace and this circumstance allows us to use all the standard tools and techniques of superspace quantum field theory.

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It is important to note that superpotential W is connected with the Kähler potential. This is because $\bar{\theta}^2 = \delta(\bar{\theta})$ is a chiral superfield and the Kähler term $\int d^4\theta\bar{\theta}^2 K(\Phi,\bar{\Phi})$ can be converted to the chiral superpotential $\int d^2\theta K(\Phi,\bar{A})$. Thus one can expect that the non-renormalization theorem for the superpotential is no longer true and actually we will see that there exist the quantum corrections to the superpotential of the deformed WZ model.[1]

The new (non)renormalization theorems have been proven [18]–[28]: the F-term is radiatively corrected and becomes indistinguishable from the D-term, while the $\bar{F}$-term is not renormalized. Since the supersymmetric vacua are critical points of the antiholomorphic superpotential, the vacuum stays stable. New divergences arise from sectors containing non-planar diagrams ([17, 18, 19, 27]). This is because the planar diagrams do not depend on the deformed parameter except those for the star product between the external legs.

The superfields $\Phi, \bar{\Phi}$ satisfy the classical equation of motion

$$\begin{aligned} D^2\Phi + \bar{m}\bar{\Phi} + \frac{1}{2}\bar{g}\bar{\Phi}^2 &= 0\,, \\ \bar{D}^2\bar{\Phi} + m\Phi + \frac{1}{2}g\Phi^2 + \frac{h}{2}(Q^2\Phi)^2 + \frac{1}{\lambda}(Q^2\Phi) &= 0\,, \end{aligned} \tag{2.3}$$

which are the algebraic equations for the auxiliary fields F and $\bar{F}$

$$-F = \bar{G} \equiv \bar{m}\bar{A} + \frac{1}{2}\bar{g}\bar{A}^2\,, \qquad -\bar{F} = G + \frac{h}{2}F^2 + \frac{1}{\lambda}F\,. \tag{2.4}$$

In component form, the effect of the star deformation leads to an additional F^3 term in the action in comparison with the standard WZ model. It means that the only scalar potential is affected by the deformation. The potential expressed in components fields is

$$V = \bar{G}\left(G + \frac{1}{\lambda}\bar{G} + \frac{h}{6}\bar{G}^2\right), \tag{2.5}$$

where $G = mA + \frac{g}{2}A^2$. By solving the equations $\frac{\partial V}{\partial A} = \frac{\partial V}{\partial \bar{A}} = 0$, $V = 0$, ones find a set of the $\mathcal{N} = 1/2$ supersymmetric vacua with vanishing vacuum energy.

It has been shown (see e.g. refs. [15, 17, 18, 26, 27, 28]) that at one-loop a divergent term proportional to F^2 appears, which is not present in the classical action. To make the theory multiplicatively renormalizable we consider a modified action with the addition of F^2 term with coupling λ in (2.2). The radiatively generated F^2-term is interesting because this the term gives rise to mass splitting between the boson and fermion component fields, while keeping the vacuum energy to zero. The appearance of F^2 divergence might lead to the conclusion that the star-product becomes deformed at the quantum level. Indeed, in ref. [26] it has been given a general argument allowing to prove that suitable resummation of such and other terms in the effective potential (in some limit of large noncommutativity parameter and small external momenta) can be rewritten in terms of open Wilson lines.

[1]Structure of the Kählerian effective potential in the standard WZ model is discussed in [29].

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3. General scheme of calculating the one-loop effective potential

In this section we describe a calculation of the one-particle irreducible (IPR) effective potential for the model (2.2). The one-loop correction to the effective action is formally written in the form

$$\Gamma_{(1)} = \frac{i}{2} \ln \mathrm{Det}(\hat{H})\,, \tag{3.1}$$

where $\hat{H}$ is the differential operator acting on superfields being the second variational derivatives over quantum (super)fields of the classical action. In order to find this operator in the framework of the loop expansion one have to split all fields of the model on quantum and background parts. We use the standard quantum-background splitting

$$\Phi \to \Phi + \varphi, \qquad \bar{\Phi} \to \bar{\Phi} + \bar{\varphi}\,,$$

where Φ and φ stand for background and quantum fields respectively. The rules for calculating variational derivatives $\varphi, \bar{\varphi}$ are analogous to the ones given in refs. [16, 30]:

$$\frac{\delta\varphi(z)}{\delta\varphi(z')} = \bar{D}^2\delta(z-z')\,, \qquad \frac{\delta\bar{\varphi}(z)}{\delta\bar{\varphi}(z')} = D^2\delta(z-z')\,.$$

We will keep both $m, \bar{m}$ nonzero so that the effective action is well defined in the infrared. The quadratic over quantum (super)fields part of the classical action is written in the form

$$S_{(2)} = \frac{1}{2}\int d^8z\, (\bar{\varphi}, \varphi)\hat{H}\begin{pmatrix} \varphi \\ \bar{\varphi} \end{pmatrix}\,, \tag{3.2}$$

where we denote

$$\hat{H} = \begin{pmatrix} D^2\bar{D}^2 & (\bar{m}+\bar{g}\bar{\Phi})D^2 \\ (m+\Lambda)\bar{D}^2 & \bar{D}^2D^2 \end{pmatrix}\,, \qquad \Lambda = g\Phi + h(Q^2\Phi)Q^2 + \frac{1}{\lambda}Q^2\,. \tag{3.3}$$

Further we use the convenient denotations

$$m + g\Phi = \mu, \qquad \bar{m} + \bar{g}\bar{\Phi} = \bar{\mu} \tag{3.4}$$

and consider the special background $\bar{\Phi} = Const$, $\bar{\mu} = Const$ which is sufficient for calculation of the chiral effective potential (see a discussion in refs. [15]–[20]).

Let's present the operator $\hat{H}$ in action $S_{(2)}$ as a sum of two operators

$$\hat{H} = \hat{H}_0 + \hat{H}_1\,, \tag{3.5}$$

where the operators are defined as follows

$$\hat{H}_0 = \begin{pmatrix} D^2\bar{D}^2 & \bar{\mu}D^2 \\ m\bar{D}^2 & \bar{D}^2D^2 \end{pmatrix}, \quad \hat{H}_0^{-1} = \begin{pmatrix} \frac{D^2\bar{D}^2}{\Box}\frac{1}{\Box-\bar{\mu}m} & \frac{-\bar{\mu}}{\Box-m\bar{\mu}}\frac{D^2}{\Box} \\ \frac{-m}{\Box-\bar{\mu}m}\frac{\bar{D}^2}{\Box} & \frac{\bar{D}^2D^2}{\Box}\frac{1}{\Box-m\bar{\mu}} \end{pmatrix}, \quad \hat{H}_1 = \begin{pmatrix} 0, & 0 \\ \Lambda\bar{D}^2 & 0 \end{pmatrix}. \tag{3.6}$$

From (3.5) using the relation $\ln(\hat{H}_0 + \hat{H}_1) = \ln(\hat{H}_0) + \ln(1 + \hat{H}_0^{-1}\hat{H}_1)$ we extract the contribution of H_0. The structure of the expression $\mathrm{Tr}\ln(\hat{H}_0)$ corresponds to an unbroken

$\mathcal{N}=1$ supersymmetry. This expression does not depend on the superfield Φ and can be omitted. As a result ones get the expression for the chiral effective potential

$$\Gamma_{(1)} = \frac{i}{2}\,\mathrm{Tr}\ln\left(1+\begin{pmatrix} \frac{-\bar{\mu}}{\Box-m\bar{\mu}}\frac{D^2\bar{D}^2}{\Box}\Lambda & 0 \\ \bar{D}^2\frac{1}{\Box-m\bar{\mu}}\Lambda & 0\end{pmatrix}\right) = \frac{i}{2}\,\mathrm{Tr}\ln\left(1-\frac{\bar{\mu}}{\Box-m\bar{\mu}}\frac{D^2\bar{D}^2}{\Box}\Lambda\right). \tag{3.7}$$

All operators $\Box, D, \bar{D}$ in this expression act trough, i.e. not only on Λ. Now we expand the logarithm and obtain a series in powers of $\frac{D^2\bar{D}^2}{\Box}\Lambda$, which can be transformed as follows

$$\frac{D^2\bar{D}^2}{\Box}\Lambda\frac{D^2\bar{D}^2}{\Box}\Lambda\cdots = \frac{D^2}{\Box}\Lambda\frac{\bar{D}^2D^2\bar{D}^2}{\Box}\Lambda\cdots = \frac{D^2}{\Box}\Lambda\bar{D}^2\Lambda\cdots = \frac{D^2\bar{D}^2}{\Box}\Lambda^2\cdots\,,$$

here we take into account that Λ is a chiral field (i.e. $[\bar{D},\Lambda]=0$) and the identity $\frac{\bar{D}^2D^2\bar{D}^2}{\Box} = \bar{D}^2$. Therefore the identity

$$\left(\frac{D^2\bar{D}^2}{\Box}\Lambda\right)^n\cdots = \frac{D^2\bar{D}^2}{\Box}\Lambda^n\cdots$$

takes place and one can write (3.7) as follows

$$\Gamma_{(1)} = \frac{i}{2}\,\mathrm{Tr}\left(\frac{D^2\bar{D}^2}{\Box}\ln\left(1-\frac{\bar{\mu}}{\Box-m\bar{\mu}}\Lambda\right)\right). \tag{3.8}$$

In order to simplify (3.8) we rewrite it in the form

$$\begin{aligned}\Gamma_{(1)} &= \frac{i}{2}\,\mathrm{Tr}\,\frac{D^2\bar{D}^2}{\Box}\ln\left(\frac{\Box-m\bar{\mu}-\bar{\mu}g\Phi-\bar{\mu}\tilde{\Lambda}}{\Box-m\bar{\mu}}\right) \\ &= \frac{i}{2}\,\mathrm{Tr}\,\frac{D^2\bar{D}^2}{\Box}\left(\ln(\Box-\mu\bar{\mu}-\bar{\mu}\tilde{\Lambda})-\ln(\Box-m\bar{\mu})\right),\end{aligned} \tag{3.9}$$

here

$$\tilde{\Lambda} = h(Q^2\Phi)Q^2+\frac{1}{\lambda}Q^2 = MQ^2\,.$$

The quantity $\ln(\Box-m\bar{\mu})$ in the second line (3.9) can be dropped out since it doesn't depend on the superfield Φ. Thus, the one-loop contribution to the effective potential (3.7) is finally presented in the following form

$$\Gamma_{(1)} = \frac{i}{2}\,\mathrm{Tr}\left(\frac{D^2\bar{D}^2}{\Box}\ln(\Box-\mu\bar{\mu}-\bar{\mu}\tilde{\Lambda})\right). \tag{3.10}$$

We pay attention on appearance of the chiral projector in this relation showing that the effective action is given by an integral over a chiral subspace. Further calculations will be fulfilled using the symbol-operator techniques [31].

3.1 Symbol-operator techniques and heat kernel representation

Here we shortly describe the basic notions of the symbol-operator techniques which is used for calculations of the one-loop effective action. The detail description of this techniques was given in papers [31]. We just remind the main stages.

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The main idea is based on the supersymmetric generalization of the well known trace formula for the operator $\hat{A} = a(\hat{q}, \hat{p})$

$$\mathrm{Tr}(\hat{A}) = \int d\mu(\gamma) A(\gamma)\,, \tag{3.11}$$

where $\hat{q}, \hat{p}$ are the operators of coordinate and momentum, $\gamma = (q, p)$ are the coordinates on the phase-space, $d\mu(\gamma)$ is a measure on the phase-space, $A(\gamma)$ is a symbol of the operator $\hat{A}$ and integration goes over the full phase-space. The symbol of the operator $\hat{A}$ is function on phase space and defined[2] as $A(\gamma) = \frac{\langle p|\hat{A}|q\rangle}{\langle p|q\rangle}$. Practical calculation of symbols is based on a relation like $A(\gamma) = a(q + i\hbar\frac{d}{dp}, p) \times 1$ where right hand side is constructed from the operator $a(\hat{q}, \hat{p})$ replacing the operator $\hat{q}$ by $q + i\hbar\frac{d}{dp}$ and the operator $\hat{p}$ by p (see e.g. [31] and the references therein).

We apply the symbol operator techniques to calculation of traces for the operators depending on $\mathcal{N} = 1$ superspace coordinates $z^M = (x^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ and corresponding derivatives. The phase superspace is parameterized by z^M, p_M where $p_M = (p_m, \psi_\alpha, \bar{\psi}_{\dot{\alpha}})$. In the case under consideration we have to consider the operators $\hat{A} = a(z, \frac{\partial}{\partial z})$. According to the symbol-operator techniques, the calculation of the trace for such operators contains the following steps (see the details in [31]):

1. We introduce the quantity $\langle p|z\rangle$ and define the symbols (z, p) of the basic operators $(z, \partial/\partial z)$ by general relation $(z, p)\langle p|z\rangle = \langle p|(\hat{z}, \hat{p})|z\rangle$.

2. To calculate a symbol of the operator function $a(z, \partial/\partial z)$ of basic operators ones replace all basic operators $z, \partial/\partial z$ by the quantities $z^\hbar, p$ respectively. The quantities $z^\hbar$ are constructed by the rule $z^\hbar = U^{-1} z U$. The appropriate operator U for $\mathcal{N} = 1$ superspace field theories is found in [31] and will be written down bellow.

3. Symbol of the operator under consideration is defined as $A(\gamma) = a(z^\hbar, p) \times 1$.

4. Trace of the operator $\hat{A}$ under consideration is given by relation (3.11).

Let us employ the above prescription for calculating the effective action (3.10). In this case the phase superspace coordinates are $\gamma = (p, \psi, \bar{\psi}; x, \theta, \bar{\theta})$, the measure in the trace definition (3.11) is $d\mu(\gamma) = d^8 z \frac{d^4 p}{(2\pi)^4} d^2\psi d^2\bar{\psi}$ and the quantity $\langle p, \psi, \bar{\psi}|x, \theta, \bar{\theta}\rangle$ is

$$\langle p, \psi, \bar{\psi}|x, \theta, \bar{\theta}\rangle = \mathrm{e}^{ip\cdot x + \theta^\alpha \cdot \psi_\alpha + \bar{\theta}^{\dot{\alpha}} \cdot \bar{\psi}_{\dot{\alpha}}}\,. \tag{3.12}$$

It allows to obtain the symbols of the supercharge Q_α and spinor derivatives $D_\alpha, \bar{D}_{\dot{\alpha}}$ in the form

$$\begin{array}{lcl} \textbf{Operator} & & \textbf{Symbol} \\ Q_\alpha = i\dfrac{\partial}{\partial\theta^\alpha} = i\partial_\alpha & \rightarrow & Q_\alpha(\gamma) = i\psi_\alpha\,, \\ D_\alpha = \partial_\alpha + \bar{\theta}^{\dot{\alpha}} i\partial_{\alpha\dot{\alpha}} & \rightarrow & D_\alpha(\gamma) = \psi_\alpha - \bar{\theta}^{\dot{\alpha}} p_{\alpha\dot{\alpha}}\,, \\ \bar{D}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}} & \rightarrow & \bar{D}_{\dot{\alpha}}(\gamma) = \bar{\psi}_{\dot{\alpha}}\,. \end{array} \tag{3.13}$$

[2]The symbol we use here is so called pq-symbol. One can show that $\mathrm{Tr}(\hat{A})$ does not depend on the choice of symbol.

In order to calculate the symbol of the operator function $a(z, \partial_m, D, \bar{D}, Q)$ we have to replace within the operator the $z, \partial/\partial z$ by the corresponding quantities $z^{\hbar}, p$ constructed with the help of a U-operator which, for the case under consideration, has the form (see the details in [31])

$$U = \mathrm{e}^{-\bar{\partial}\bar{D}} \mathrm{e}^{\partial p \cdot \bar{\theta}} \mathrm{e}^{-\partial D} \mathrm{e}^{-i\partial_p \partial_x}, \tag{3.14}$$

where $\partial^{\alpha} = \frac{\partial}{\partial \psi_{\alpha}}$, $\bar{\partial}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\psi}_{\dot{\alpha}}}$ and the derivatives D, $\bar{D}$, ∂_z act on the left while the operators $\frac{\partial}{\partial \psi}$, $\frac{\partial}{\partial \bar{\psi}}$, ∂_p act on the right. As a result ones get the operator acting in phase superspace[3]

$$A^{\hbar} = a(z^{\hbar}, p^{\hbar}, D^{\hbar}, \bar{D}^{\hbar}, Q^{\hbar})\,. \tag{3.15}$$

Simple calculations lead to

$$\bar{D}^{\hbar}_{\dot{\alpha}} = \bar{\psi}_{\dot{\alpha}}\,, \qquad D^{\hbar}_{\alpha} = \psi_{\alpha} - \bar{\partial}^{\dot{\alpha}} p_{\alpha\dot{\alpha}}\,, \qquad Q^{\hbar}_{\alpha} = i(\psi_{\alpha} + p_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}})\,, \qquad p^{\hbar}_{\alpha\dot{\alpha}} = p_{\alpha\dot{\alpha}}\,. \tag{3.16}$$

These operators obey the initial algebra but act on functions of phase superspace coordinates. For the background field one can get the representation

$$\Phi^{\hbar}(y, \theta) = \Phi(y + i\partial_p, \theta + \partial_{\psi}) = \Phi + \partial^{\alpha}(D_{\alpha}\Phi) - \partial^2(D^2\Phi) + \mathcal{O}(\partial_y)\,, \tag{3.17}$$

here the derivatives $\partial = \partial/\partial\psi$ act through on the right. The terms including the y-derivatives of Φ can be omitted because we use the approximation of background fields slowly varying in space-time.

Now we consider the effective action (3.10). It is written as a trace of the operator depending on $z, D.\bar{D}, Q$ and hence, we can apply to its calculation the symbol operator techniques we shortly described above. After replacement in (3.10) the derivatives and fields by corresponding quantities (3.16) and (3.17) we obtain the representation for the one-loop effective action in the form:

$$\Gamma_{(1)} = \frac{i}{2}\int d^8 z \int d^2\psi d^2\bar{\psi}\, \frac{d^4 p}{(2\pi)^4} \left(\frac{\psi^2\bar{\psi}^2}{-p^2}\right) \ln(-p^2 - \bar{\mu}\cdot(m + g\Phi^{\hbar}(y,\theta) + M(Q^{\hbar})^2)) \tag{3.18}$$

$$M = h(Q^2\Phi) + \frac{1}{\lambda}\,. \tag{3.19}$$

We replace in (3.10) the operator $\frac{D^2\bar{D}^2}{\Box}$ with (3.16) and take into account that the non-zero integral over $\bar{\psi}$ must contain the factor $\bar{\psi}^2$. Since such a factor is completely formed in $\bar{D}^2_{\hbar}$ and saturates the integral over $\bar{\psi}$, we can drop $\bar{\partial}^{\dot{\alpha}} p_{\alpha\dot{\alpha}}$ in $D^2_{\hbar}$.

Last step of calculations is to act on unit in (3.18). It is equivalent to moving all derivatives $\partial_{\psi}, \partial_{\bar{\psi}}$ in (3.18) on the right and drop them. The properties of the superspace integration and differentiation

$$\int d^2\psi\, A \equiv \partial^2 A\,, \qquad \partial^2\cdot\psi^2 = -1 \tag{3.20}$$

can be used in (3.18) for calculation.

[3]Index $\hbar$ marks the quantities transformed with the help of U-operator.

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Useful elements of calculations of the effective action under consideration is ζ-function representation of eq. (3.18). To find it we use proper-time integral representation of the logarithm. It is convenient also to introduce a dimension parameter L^2 which serves for compensation of the dimensional quantity in the proper-time exponent.[4] With the mentioned notations the ζ-function representation is

$$\begin{aligned}\Gamma_{(1)} &= \left(-\frac{d}{ds}\right)\Big|_{s=0} \frac{i}{2}\int d^8z \int \frac{d^4p}{(2\pi)^4}\int d^2\psi d^2\bar{\psi}\left(\frac{\psi^2\bar{\psi}^2}{-p^2}\right)\times \\ &\quad\times \int_0^\infty \frac{dT}{\Gamma(s)}T^{s-1}(L^2)^s e^{-T(p^2+\bar{\mu}\mu+\bar{\mu}\tilde{\Lambda})} \\ &= \left(-\frac{d}{ds}\right)\Big|_{s=0}\int_0^\infty \frac{dT}{\Gamma(s)}\,T^{s-1}\int d^6z\,\bar{D}^2K\left(\frac{T}{L^2}\right), \end{aligned} \tag{3.21}$$

where the heat kernel in the above equation is defined as

$$K(T) = \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\,\frac{1}{-p^2}\left(\frac{L^2}{\bar{\mu}}\right)^s \bar{\mu}\,\mathrm{e}^{-T(p^2+m)}\mathrm{e}^{-T(MQ_\hbar^2+g\Phi_\hbar(y,\theta))}\times 1\,. \tag{3.22}$$

It should be noted that our aim is study only the chiral effective potential and, therefore, we rewrite the integral over grassmannian variables in (3.21) via a chiral integral.

In (3.21), for simplicity, we changed the denotation for variables:

$$\frac{p^2}{\bar{\mu}} \to p^2\,, \qquad T\bar{\mu} \to T\,. \tag{3.23}$$

In further calculation we have to keep in mind these replacement.

As it has been shown by direct calculation in refs. [8, 10, 11, 19, 20] and presented as a theorem in [27] the chiral superpotential in $\mathcal{N}=1/2$ model is proportional to $\bar{\theta}^2$. The presence of the term $\sim \Phi(Q^2\Phi)^2$ in the classical action (2.2) leads to possible action of Q^2 inside the loop integrals, and this results effectively in $\bar{\theta}^2$-dependence.

The general structure of the effective action in $\mathcal{N}=1/2$ supersymmetry models is given as [27]

$$\Gamma[\Phi,\bar{\Phi}] = \sum_n \int \prod_{j=1}^n d^4x_j \int d^2\theta d^2\bar{\theta}\,G(x_1,\ldots,x_n;\bar{\theta}^2)F_1(x_1,\theta,\bar{\theta})\cdots F_n(x_n,\theta,\bar{\theta})\,, \tag{3.24}$$

where $G(x_1,\ldots,x_n;\bar{\theta}^2)$ are translation-invariant functions of coordinates x and possible insertion $\bar{\theta}^2$, while $F(x,\theta,\bar{\theta})$ are local operators of $\Phi,\bar{\Phi}$, their covariant derivatives, and the expressions $Q\Phi$. The insertion of $\bar{\theta}^2$ from $G(x_1,\ldots,x_n;\bar{\theta}^2)$ absorbs the $\int d^2\bar{\theta}$ integral. Because of this, the D-terms with pure chiral fields and holomorphic F-terms coincide between each other and, therefore, both D-terms and F-terms are unified in $\mathcal{N}=1/2$ supersymmetry [27].

As a result, the integral over full superspace (3.21) is transformed to the integral over chiral subspace $\int d^8z\bar{\theta}^2\cdots = \int d^6z\bar{D}^2\bar{\theta}^2\cdots$. Therefore quantity $\bar{\mu}$ will contribute to the effective action (3.22) only one of it's component $\bar{\mu}=\bar{m}+\bar{g}\bar{A}$ Using equation of motion it means $\bar{\mu}^2=\bar{m}^2-2\bar{g}F$. Thus, the expression for chiral potential (3.21) does not contain any antichiral superfields.

[4]The proper time T became dimensionless.

3.2 Exact calculation of the heat kernel

In the present section we present a method allowing to find an exact expression for the heat kernel. In general, exact calculation of the heat kernel is impossible. The model under consideration is quite remarkable since it provides the exact evaluation of the heat kernel. The reason is the fact that for this model the heat kernel calculation is reduced to finding an evolution operator for a harmonical oscillator with the grassmannian coordinate ψ and momentum $\partial/\partial\psi$.

In order to calculate (3.22), according to the symbol-operator techniques, we have to disentangle derivatives in the exponent of the heat kernel. To do that we transfer all derivatives ∂_ψ on the right and act on unit. It means that after such a transformation all terms with derivatives must be omitted and a final contribution is resulted only from recommutations of the differential operators to the last right position. The rest part is the symbol of the heat kernel. Finally, the trace of hear kernel is given by integration of the corresponding symbols over the phase space (see e.g. [31]).

Let's denote the right exponent in (3.22) as

$$h(T) = \mathrm{e}^{-T(MQ_\hbar^2 + g\Phi_\hbar(y,\theta))} \times 1\,. \tag{3.25}$$

Using (3.17) one can rewrite it in the form

$$h(T) = \mathrm{e}^{TM\tilde{Q}_\hbar^2 - Tg\Phi - Tg\partial^\alpha(D_\alpha\Phi) + Tg\partial^2(D^2\Phi)} \tag{3.26}$$

where $\partial^\alpha = \frac{\partial}{\partial\psi_\alpha}$, $\tilde{Q}^\hbar_\alpha = (\psi + \sqrt{\bar{\mu}}p\bar{\theta})_\alpha$ and M was defined in (3.19).

The expression in the exponent (3.26) can be simplified by introducing a new denotation $\tilde{\partial}_\alpha = \partial_\alpha - \frac{D_\alpha\Phi}{D^2\Phi}$, then we can extract from (3.26) ψ- and ∂_ψ-independent part

$$h(T) = e^{-Tg\Phi - Tg\frac{(D\Phi)^2}{D^2\Phi}} \cdot k(T) \times 1\,, \qquad k(T) = \mathrm{e}^{TM\tilde{Q}_\hbar^2 + Tg(D^2\Phi)\tilde{\partial}^2}\,. \tag{3.27}$$

Straightforward calculations of the commutation relations between operators in operator-dependent part $k(T)$ lead to the following algebra

$$\{\tilde{\partial}^\alpha, \tilde{Q}^\hbar_\beta\} = \delta^\alpha_\beta \qquad [\tilde{\partial}^2, \tilde{Q}^2_\hbar] = (\tilde{Q}^\hbar_\alpha\tilde{\partial}^\alpha - 1)\,, \tag{3.28}$$

i.e. $\tilde{Q}^\hbar_\beta$ and $\tilde{\partial}^\alpha$ can be considered as the Grassmann coordinates and momenta.

We will show that the function $k(T) \times 1$ can be calculated exactly. It means that the whole heat kernel in (3.22) is exactly found. We pay attention that the operator $k(T) = \mathrm{e}^{TM\tilde{Q}_\hbar^2 + Tg(D^2\Phi)\tilde{\partial}^2}$ can be evidently treated as an evolution operator for a quantum system with hamiltonian $\mathcal{H} = -(M\tilde{Q}^2_\hbar + g(D^2\Phi)\tilde{\partial}^2)$. This hamiltonian is a quadratic form in grassmannian coordinates ψ and corresponding momenta $\partial/\partial\psi$ with some coefficients. Such a quantum system is called the grassmannian oscillator (see e.g. [32]). As a result, the problem under consideration is reduced to a problem of finding the trace for the evolution operator of the grassmannian oscillator which can be solved exactly on the base of its algebraic properties.

Let us introduce denotations for the operators

$$e_1 = \tilde{Q}^2_\hbar\,, \qquad e_2 = \tilde{\partial}^2\,, \qquad e_3 = \tilde{Q}^\hbar_\alpha\tilde{\partial}^\alpha - 1\,. \tag{3.29}$$

It is easy to see that these operators satisfy the commutation relations for the generators of $su(2)$ algebra

$$[e_1, e_2] = e_3\,, \qquad [e_3, e_1] = 2e_1\,, \qquad [e_3, e_2] = -2e_2\,. \tag{3.30}$$

Hence, the exponent in $k(T)$ (3.27) is nothing but a group element of SU(2)

$$k(T) = \mathrm{e}^{TMe_1 + TgFe_2}\,, \tag{3.31}$$

where

$$F = D^2\Phi, M = h(Q^2\Phi) + \frac{1}{\lambda}\,. \tag{3.32}$$

This observation allows to simplify all further consideration.

Since our goal is to find a symbol of the heat kernel, we should move all derivatives in the exponent (3.27) to right hand side and act on unit what is equivalent to drop them. The generators containing the derivatives in the group element $k(T)$ are e_2, e_3. It is most convenient to rewrite the group element $k(T)$ in the gaussian form

$$k(T) = \mathrm{e}^{TMe_1 + TFe_2} = \mathrm{e}^{A(T)e_1}\mathrm{e}^{B(T)e_3}\mathrm{e}^{C(T)e_2}\,. \tag{3.33}$$

To find functions A, B, C we calculate $\frac{dk(T)}{dT} \cdot k^{-1}(T)$ for first $k = \mathrm{e}^{TMe_1+TFe_2}$ and for second $k = \mathrm{e}^{A(T)e_1}\mathrm{e}^{B(T)e_3}\mathrm{e}^{C(T)e_2}$ representations for k from the expression (3.33). Because the results must be equal, it leads to constrains on the coefficients

$$Me_1 + Fe_2 = e_1(\dot{A} - 2A\dot{B} - A^2\dot{C}e^{-2B}) + e_2\dot{C}e^{-2B} + e_3(\dot{B} + A\dot{C}e^{-2B})\,. \tag{3.34}$$

Comparison of the coefficients at each e_i leads to a set of first order differential equations for A, B, C. In particular, one of these equations looks like $\dot{A} + A^2F = M$ and has the solution

$$A = \sqrt{\frac{M}{F}}\tanh(T\sqrt{MF})\,. \tag{3.35}$$

The other equations have the solutions

$$B = -\ln\cosh(T\sqrt{FM})\,, \qquad C = \sqrt{\frac{F}{M}}\tanh(T\sqrt{FM})\,. \tag{3.36}$$

Here the F and M are given by (3.32). Using definition of the generators (3.29) and dropping the derivatives acting on unit in (3.33) one obtains

$$\begin{aligned}
\mathrm{e}^{A(T)e_1} &= \mathrm{e}^{AQ^2}\,,\\
\mathrm{e}^{B(T)e_3} &= \mathrm{e}^{-B(T)}\exp\left((1 - e^{B(T)})Q_\alpha\frac{gD^\alpha\Phi}{F}\right),\\
\mathrm{e}^{C(T)e_2}\times 1 &= \mathrm{e}^{C(T)\tilde{\partial}^2} = \exp\left(C(T)g^2\frac{D^\alpha\Phi D_\alpha\Phi}{2F^2}\right),
\end{aligned} \tag{3.37}$$

where $Q_\alpha = \sqrt{\bar{\mu}}p_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$, $Q^2 = -\bar{\mu}p^2\bar{\theta}^2$ due to (3.16), (3.23) and only terms without $\psi, \bar{\psi}$ are kept because the definition (3.21) already contains the factor $\psi^2\bar{\psi}^2$.

As a result one obtains for the symbol (3.25) the following expression

$$k(T)\times 1 = Q^2\left(A(T)+(1-e^B)^2\frac{g^2}{F^2}(D\Phi)^2\right)\exp\left(-B(T)+C(T)\frac{g^2}{F^2}(D\Phi)^2\right), \qquad (3.38)$$

where $(D\Phi)^2=\frac{1}{2}D^\alpha\Phi D_\alpha\Phi$ and the $A(T),B(T),C(T)$ are given by (3.35), (3.36)

Now using (3.27), (3.38) in (3.21), (3.22) we obtain the exact expression for the one-loop chiral potential

$$\Gamma_{(1)} = -\left.\frac{d}{ds}\right|_{s=0}\int d^6z\int_0^\infty \frac{dT}{\Gamma(s)}T^{s-1}\frac{1}{2(4\pi T)^2}\left(\frac{L^2}{\bar\mu}\right)^s\bar\mu^2\mathrm{e}^{-T(m+g\Phi)}\cdot\tilde{k}(T)\,, \qquad (3.39)$$

where

$$\tilde{k}(T) = \left(\sqrt{\frac{M}{F}}\tanh(T\sqrt{FM})+\left(1-\frac{1}{\cosh(T\sqrt{FM})}\right)^2\frac{g^2}{F^2}(D\Phi)^2\right)\times$$
$$\times\cosh(T\sqrt{FM})\exp\left(Tg^2\frac{(D\Phi)^2}{F}\left(\frac{\tanh(T\sqrt{FM})}{T\sqrt{FM}}-1\right)\right) \qquad (3.40)$$

can be further simplified. Taking into account the grassmannian property $(D\Phi)^3=0$ and expanding the exponent in (3.40) one can find

$$\tilde{k}(T)=\sinh(T\sqrt{FM})\left(\sqrt{\frac{M}{F}}+T\sqrt{FM}\left(\frac{\tanh(T\sqrt{FM}/2)}{T\sqrt{FM}/2}-1\right)\frac{g^2}{F^2}(D\Phi)^2\right). \qquad (3.41)$$

Now we take into account the property $\int D^\alpha(\Phi D_\alpha\Phi e^{-Tg\Phi})=0$, which can be used in (3.39). We get $(D\Phi)^2=-\frac{\Phi F}{1-Tg\Phi}$. As a result one obtains the expression

$$\tilde{k}(T)=\sqrt{\frac{M}{F}}\sinh(T\sqrt{FM})\left(1-g\frac{Tg\Phi}{1-Tg\Phi}\left(\frac{\tanh(T\sqrt{FM}/2)}{T\sqrt{FM}/2}-1\right)\right). \qquad (3.42)$$

The expressions (3.39), (3.42) determine the final exact solution for the one-loop chiral effective potential in $\mathcal{N}=1/2$ WZ model. The various approximate results can be obtained using the various expansions of (3.42). Also we point out that the integral (3.39) is divergent at the low limit. To get a finite effective potential we should, as usual, to subtract in the integrand of (3.42) a first term in expansion of the integrand in T. We will specially discuss a structure of divergences in the theory under consideration in subsection 4.1.

3.3 Expansion of the heat kernel

The exact result for the one-loop chiral effective potential is presented by the expressions (3.39), (3.42) in the form of an integral over proper time T which can not be written in an explicit form in terms of elementary or known special functions. To obtain the various approximate results we have to construct the expansions of the heat kernel and calculate the integral over proper time in an explicit form. In this section we formulate some independent procedure for the heat kernel expansion which doesn't use (3.39) at all and based on the Fourier transforms in the grassmannian variables. It allows us to develop an approximate scheme for calculating the chiral effective potential in a form of a power expansion of spinor derivatives of Φ. Of course, such an expansion can be constructed, in principle, on the base of exact result (3.39), (3.42). However, technically it is much more easy to begin with initial relation (3.22).

Let's present the exponent (3.25) as a sum

$$h(T) = e^{T(H_0+V)} \times 1 = \sum_{n=0}^{\infty} h_n(T) \times 1\,, \quad h_0 = \mathrm{e}^{TH_0}\,, \quad H_0 = -MQ_\hbar^2\,, \quad V = -g\Phi_\hbar(y,\theta)\,, \tag{3.43}$$

where the general term of the sum is given by the T-ordered iterated integral

$$h_n(T) \times 1 = \int_0^T dt_n \int_0^{t_n} dt_{n-1} \ldots \int_0^{t_2} dt_1\, \mathrm{e}^{(T-t_n)H_0} V \mathrm{e}^{(t_n-t_{n-1})H_0} V \ldots V \mathrm{e}^{(t_2-t_1)H_0} V \mathrm{e}^{t_1 H_0}\,, \tag{3.44}$$

(see details in ref. [33]). This integral for every fixed n can be calculated. Firstly, we make a replacement of the variables

$$\begin{cases} s_1 = t_2 - t_1\,, \\ \quad\vdots \\ s_{n-1} = t_n - t_{n-1}\,, \\ t = T - t_n\,, \end{cases} \tag{3.45}$$

using the rules $(t_1, \ldots, t_n) \to (s_1, \ldots, s_{n-1}, t)$. This replacement does not change integration because $\frac{\partial(s_1,\ldots,s_n,t)}{\partial(t_1,\ldots,t_n)} = 1$ and

$$\sum_{i=1}^{n-1} s_i \le T - t \le T\,, \qquad 0 \le s_i\,, \qquad i = 1, \ldots, n-1\,.$$

Therefore one can write the integral in (3.44) as follows

$$\int_0^T dt_n \int_0^{t_n} dt_{n-1} \cdots \int_0^{t_2} dt_1 = \int ds_1 \cdots ds_{n-1} \int_0^{T-\sum_{i=1}^{n-1} s_i} dt\,; \qquad 0 \le s_i\,, \quad \sum_{i=1}^{n-1} s_i \le T\,.$$

Let's introduce a redundant variable $s_n = T - \sum_{i=1}^{n-1} s_i$, then the general term (3.44) can be written in a symmetrical form

$$h_n(T) \times 1 = \int d^n s\, \delta\left(T - \sum_{i=1}^{n} s_i\right) \int_0^{s_n} ds_{n+1}\ \mathrm{Tr}\left(\mathrm{e}^{tH_0} V \mathrm{e}^{s_1 H_0} \cdots V \mathrm{e}^{s_{n-1} H_0} V \mathrm{e}^{s_n H_0} \mathrm{e}^{-tH_0}\right). \tag{3.46}$$

Because

$$\int_0^{s_n} ds_{n+1} = s_n = \frac{1}{n} \sum_{i=1}^{n} s_i = \frac{1}{n} T\,,$$

one can also write

$$h_n(T) \times 1 = \frac{T}{n} \int d^n s\, \delta\left(T - \sum_{i=1}^{n} s_i\right) \mathrm{Tr}\left(V e^{s_1 H_0} \cdots V e^{s_n H_0}\right). \tag{3.47}$$

For further calculation we use another replacement of the variables $s_i \to \alpha_i$, $\alpha_i = s_i/T$ in (3.43) along with the Fourier transform for the grassmannian variables

$$V(y,\theta) = -g\Phi_\hbar(y,\theta) = \int d^2\pi\,(-g\Phi(y,\pi))\,e^{(\theta+\partial)\pi}\,, \qquad \partial = \frac{\partial}{\partial\psi}\,, \tag{3.48}$$

where the Fourier transform of the background field is

$$\Phi(y,\pi) = -\int d^2\theta\,\mathrm{e}^{-\theta\pi}\,\Phi(y,\theta) = F(y) + \pi^\alpha\kappa_\alpha(y) - \pi^2 A(y)\,. \tag{3.49}$$

This transformation allows to rewrite the expression (3.47) in the form

$$\begin{aligned} h_n(T)\times 1 = &\int \prod_{i=1}^{n} d^2\pi_i\,[-g\Phi(\pi_i)]\frac{T^n}{n}\times \\ &\times \int_{\alpha_i\geq 0} d^n\alpha\,\delta\left(1-\sum^{n}\alpha_i\right)\mathrm{e}^{\theta(\sum\pi_i)}E_{(n)}(\alpha_1,\ldots\alpha_n) \end{aligned} \tag{3.50}$$

where we introduced the new denotation

$$E_{(n)}(\alpha_1,\ldots\alpha_n) = \mathrm{e}^{\partial\pi_1}\mathrm{e}^{T\alpha_1 H_0}\mathrm{e}^{\partial\pi_2}\mathrm{e}^{T\alpha_2 H_0}\times\cdots\times\mathrm{e}^{\partial\pi_n}\mathrm{e}^{T\alpha_n H_0}\times 1\,. \tag{3.51}$$

The last expression can be simplified. Because $TH_0 = -TMQ_\hbar^2$ it is useful temporarily introduce a new variable $u = -TM$. Using the relation

$$\mathrm{e}^{\partial\pi}\mathrm{e}^{\alpha u Q_\hbar^2}\mathrm{e}^{-\partial\pi}\mathrm{e}^{\partial\pi} = \mathrm{e}^{\alpha u(Q_\hbar^2 - i\pi Q_\hbar - \pi^2)}\mathrm{e}^{\partial\pi}\,, \tag{3.52}$$

where $\pi^2 = \frac{1}{2}\pi^\alpha\pi_\alpha$ along with the property $e^{\partial\pi}\times 1 = 1$, we transform the exponent sequence in (3.51) to

$$\mathrm{e}^{\alpha_1 u(Q^2 - i\pi_1 Q - \pi_1^2)}\mathrm{e}^{\alpha_2 u(Q_\hbar^2 - i(\pi_1+\pi_2)Q_\hbar - (\pi_1+\pi_2)^2)}\times\cdots\times\mathrm{e}^{\alpha_n u(Q_\hbar^2 - i(\pi_1+\cdots+\pi_n)Q_\hbar - (\pi_1+\cdots+\pi_n)^2)}\,. \tag{3.53}$$

Collecting terms with the same powers $Q_\hbar$ one can write

$$\begin{aligned} E_{(n)}(\alpha_1,\ldots\alpha_n) = \exp\Big(&Q_\hbar^2 u(\alpha_1+\cdots+\alpha_n) - \\ &-[\alpha_1\pi_1 + \alpha_2(\pi_1+\pi_2) + \cdots + \alpha_n(\pi_1+\cdots+\pi_n)]iuQ_\hbar - \\ &- u[\alpha_1\pi_1^2 + \alpha_2(\pi_1+\pi_2)^2 + \cdots + \alpha_n(\pi_1+\cdots+\pi_n)^2]\Big)\times 1\,. \end{aligned} \tag{3.54}$$

For the integration over grassmannian variables ψ in (3.21) we must keep in $E_{(n)}$ only terms which don't contain ψ at all. That means that we can drop all ψ in further calculation. Because we made changes of variables (3.23) the expression for supercharge now is $Q_\hbar = i\sqrt{\bar\mu}\hat p\cdot\bar\theta$. Using relation $e^{u(Q_\hbar^2 - i\Gamma^\alpha Q_\alpha^\hbar)} = e^{u\tilde Q_\hbar^2}e^{u\Gamma^2}$, new denotations $\tilde Q = Q_\hbar - i\Gamma$, $Q_\hbar^2 = -\bar\mu p^2\bar\theta^2$ and properties

$$\bar D^2\tilde Q^2| = \bar\mu p^2\,, \qquad \int\frac{d^4p}{(2\pi)^4}e^{-Tp^2} = \frac{i}{(4\pi T)^2}\,, \tag{3.55}$$

we transfer the consequence (3.54) to a final form

$$E_{(n)}(\alpha_1,\dots\alpha_n) = \exp\Big(TM\left[\alpha_1\pi_1^2+\alpha_2(\pi_1+\pi_2)^2\cdots+\alpha_n(\pi_1+\ldots+\pi_n)^2\right]+ \\ -TM\left[\alpha_1\pi_1+\alpha_2(\pi_1+\pi_2)+\ldots+\alpha_n(\pi_1+\cdots+\pi_n)\right]^2\Big)\,, \quad (3.56)$$

where $u=-TM$. This leads to the following representation for the one-loop chiral effective potential

$$\Gamma_{(1)} = \left(-\frac{d}{ds}\right)\Big|\int d^4x\int d^2\theta\int_0^\infty \frac{dT}{\Gamma(s)}\frac{T^{s-1}}{2(4\pi T)^2}(-TM)\left(\frac{L^2}{\bar\mu}\right)^s\bar\mu^2\mathrm{e}^{-Tm}\times \\ \times\sum_{n=0}^{\infty}\int\prod_{i=1}^{n}d^2\pi_i[-g\Phi(y,\pi_i)]\frac{T^n}{n}\,\mathrm{e}^{\theta(\sum_{i=1}^n\pi_i)}\int_{\alpha_i\geq 0}d^n\alpha\times \\ \times\delta\left(1-\sum^n\alpha_i\right)E_{(n)}(\alpha_1,\dots\alpha_n)\,. \quad (3.57)$$

This is the basic relation allowing to develop a scheme for approximate calculations of the chiral effective potential.

Thus, we obtained the one-loop chiral effective potential in two forms, in form of exact integral representation (3.39) and in form of expansion (3.57). Integral representation (3.39) has independent significance and is one of our main results here. The expansion (3.57) will be used further for obtaining the various approximate results for the chiral effective potential.

4. Evaluations of the chiral effective potential

Now we evaluate the chiral effective potential for the model (2.2) using the representation (3.57) and the quantities $E_{(n)}$ which appeared in the heat kernel representation (3.50). It is clear that in general the chiral effective potential should have the divergent and the finite parts

$$\Gamma_{(1)} = \Gamma^{\rm div}_{(1)}+\Gamma^{\rm fin}_{(1)} = \int d^6z\,(W_{\rm div}(\Phi,D^2\Phi)+W_{\rm fin}(\Phi,D^2\Phi))\,, \quad (4.1)$$

where $W_{\rm div}$ is a divergent part which is discussed in the section 4.1, the finite part $W_{\rm fin}$ is studied in the sections 4.2, 4.3.

Because π are the grassmannian momenta $\pi^3=0$, the quantities $E_{(n)}$ are the finite polynomials in π_i for each fixed n in (3.57). Below we will demonstrate the calculation of several $E_{(n)}$ in an explicit form. It is worth pointing out that expression (3.57) has a structure of the well-known analytical representation for the one-loop Feynman integrals and obviously it could be found summing up the contributions of all one-loop diagrams with arbitrary number of external legs.

Let us present (3.57) in the following form

$$\Gamma_{(1)} = \sum_{n=0}^{\infty}R_n\,, \quad (4.2)$$

where

$$
\begin{aligned}
R_{(n)} = & \left(-\frac{d}{ds}\right)\bigg| \int d^4x \int d^2\theta \int_0^\infty \frac{dT}{\Gamma(s)} \frac{T^{s-1}}{2(4\pi T)^2}(-TM)\left(\frac{L^2}{\bar{\mu}}\right)^s \bar{\mu}^2 \mathrm{e}^{-Tm} \times \\
& \times \int \prod_{i=1}^n d^2\pi_i[-g\Phi(y,\pi_i)] \frac{T^n}{n} \mathrm{e}^{\theta(\sum_{i=1}^n \pi_i)} \int_{\alpha_i \geq 0} d^n\alpha \times \\
& \times \delta\left(1 - \sum^n \alpha_i\right) E_{(n)}(\alpha_1, \ldots \alpha_n)\,.
\end{aligned} \tag{4.3}
$$

Further it will be shown that relation (4.2) leads to the effective potential in the form of expansion in power in $D^2\Phi$ with coefficients dependent on Φ

$$
\Gamma_{(1)} = \int d^4x d^2\theta\, W_{\text{eff}}(\Phi, D^2\Phi)\,, \qquad W_{\text{eff}}(\Phi, D^2\Phi) = \sum_{k=0}^\infty W_k(\Phi)(D^2\Phi)^k\,, \tag{4.4}
$$

where $W_k(\Phi)$ is an infinite series over powers of Φ which goes from all R_l with $l > k$.

In is worth paying attention again on the expansion (4.2) which is nothing but an ordinary diagram expansion: each R_n corresponds to a contribution from diagram having n-legs Φ on the background of constant $D^2\Phi$. It means that the corresponding contributions contain the mentioned constant background dependence. The operator techniques we use here allows to obtain this expansion by simpler way. A sense of the expansion (4.2) can be traced from analysis of eq. (3.57). According to eq. (3.44) the quantity $E_{(n)}$ defines the heat kernel expansion. The sum over n in (3.57) appeared from the heat kernel expansion (3.43), (3.50). From eq. (3.44) it is follows that the contribution from h_n for any fixed n goes from n insertion of the background field V (or Φ) (3.48). It means that a term R_n in the expansion (4.2) could correspond to the contribution from the diagram having n external legs. Nevertheless, according to (3.4), (3.18) the external field Φ is also presented in μ and in M, which are contained in the propagator. The quantity M consists of two terms: first term $h(Q^2\Phi)$ will insert additional background field, while the second term $1/\lambda$ will not. Finally we can conclude that the term R_n in the expansion (4.2) consists from the contributions of several one-loop diagrams: namely from diagrams having $n, n+1, \ldots, 2n$ external fields Φ.

As one can see from (3.56) for practical purposes it is useful to introduce the new variables (linear combinations of the external momenta)

$$
\begin{aligned}
l_1 &= \pi_1\,, \\
l_2 &= \pi_1 + \pi_2\,, \\
&\vdots \\
l_n &= \pi_1 + \cdots + \pi_n\,, \\
l_{ij} &= l_i - l_j\,.
\end{aligned} \tag{4.5}
$$

The inverse transformation is

$$
\pi_1 = l_1\,,
$$

$$
\begin{aligned}
\pi_2 &= l_2 - l_1 = l_{21}\,, \\
\pi_3 &= l_3 - l_2 = l_{32}\,, \\
&\vdots \\
\pi_n &= l_n - l_{n-1} = l_{n\,n-1}\,,
\end{aligned} \tag{4.6}
$$

and, therefore, the above transformation leads in (3.57) to $\Phi(\pi_k) \to \Phi(l_{k\,k-1})$. Using property $\sum_i \alpha_i = 1$, in terms of variables (4.5), the exponent (3.56) can be written as

$$
E_{(n)}(\alpha_1, \ldots \alpha_n) = \exp(TM(\underbrace{\alpha_1\alpha_2 l_{12}^2 + \alpha_1\alpha_3 l_{13}^2 + \ldots + \alpha_1\alpha_n l_{1n}^2 + \cdots + \alpha_{n-1}\alpha_n l_{n-1\,n}^2}_{n(n-1)/2}))\,. \tag{4.7}
$$

Because π_i are grassmannian momenta, $(l_{i_1i_2}^2)^k = 0$ for any $k > 1$, the expansion of the exponent for fixed n will lead to a finite polynomial in π_i. The expansion of the above exponential argument gives $\frac{n(n-1)}{2}$ terms with different coefficients $\alpha_{i_1}\alpha_{i_2}$

$$
E_{(n)} = 1 + \sum_{k=0}^{n} \frac{1}{k!} \sum_j (k; j_1, \ldots, j_{\frac{n(n-1)}{2}}) \prod_{i=1}^{\frac{n(n-1)}{2}} (TM\tilde{\alpha}_i l_i^2)^{j_i}\,, \tag{4.8}
$$

where $\sum_j$ goes over all possible combinations $j_i = 0, 1$ such that $\sum_i j_i = k$ and $i = 1, \ldots, \frac{n(n-1)}{2}$; $(k; j_1, \ldots j_m)$ is the number of ways to put $k = j_1 + \ldots + j_m$ different things into m different boxes and $j_i = 0, 1$; $l_i = l_{i_1i_2}$ is the variables (4.6); $\tilde{\alpha}_i = \alpha_{i_1}\alpha_{i_2}$ with the same induces as the corresponding $l_{i_1i_2}$.

For example

$$
\begin{aligned}
E_{(2)} &= \exp(TM\alpha_1\alpha_2 l_{12}^2) = 1 + \frac{1}{2} TM\alpha_1\alpha_2\pi_1\pi_2\,, \\
E_{(3)} &= \exp(TM(\alpha_1\alpha_2 l_{12}^2 + \alpha_1\alpha_3 l_{13}^2 + \alpha_2\alpha_3 l_{23}^2)) \\
&= 1 + TM(\alpha_1\alpha_2 l_{12}^2 + \alpha_1\alpha_3 l_{13}^2 + \alpha_2\alpha_3 l_{23}^2) + \\
&\quad + (TM)^2(\alpha_1^2\alpha_2\alpha_3 l_{12}^2 l_{13}^2 + \alpha_1\alpha_2^2\alpha_3 l_{12}^2 l_{23}^2 + \alpha_1\alpha_2\alpha_3^2 l_{13}^2 l_{23}^2) + \\
&\quad + (TM)^3 \alpha_1^2\alpha_2^2\alpha_3^2 l_{12}^2 l_{13}^2 l_{23}^2\,,
\end{aligned} \tag{4.9}
$$

where $l_{12}^2 = \pi_2^2 = -\frac{1}{2}\pi_2(\pi_1 + \pi_3)$, $l_{13}^2 = \pi_1^2 = -\frac{1}{2}\pi_1(\pi_2 + \pi_3)$, $l_{23}^2 = \pi_3^2 = -\frac{1}{2}\pi_3(\pi_1 + \pi_2)$.

One can note that the expansions (4.8) correspond to the derivative expansion in the coordinate space. All terms in (4.8), excluding the first one, contain the momenta π, which after the inverse Fourier transform are similar to the one given in eq. (3.48) will correspond to a grassmannian derivation, i.e. $\pi_i\Phi_i \to D\Phi$. The first term in (4.8) doesn't contain π and therefore leads to the contribution without derivatives.

After expansion of the $E_{(n)}$ in a series, the finding of R_n contribution in (4.2) consists in calculation of the proper-time and α- integrals. The integrals over alphas can be calculated using expression

$$
\int \prod_i^n d\alpha_i\, \delta\left(1 - \sum_i^n \alpha_i\right) \alpha_1^{j_1}\alpha_2^{j_2}\cdots\alpha_n^{j_n} = \frac{\Gamma(j_1+1)\Gamma(j_2+1)\cdots\Gamma(j_n+1)}{\Gamma(j_1+j_2+\cdots+j_n+n)}\,. \tag{4.10}
$$

It can be also useful to fulfil the Fourier transform inverse to (3.49), which is equivalent to the replacement $\pi_i\Phi_i \to D\Phi$. It is easy to see that the structure of a general element of the expansion contains $\Phi^m(D^2\Phi)^n$. Such a structure belongs to the ring structure which corresponds to the ring structure discussed in [34].

In the following sections we will show that for the model under consideration any term R_n can be expressed via hypergeometric functions of n variables. Due to grassmannian properties of the variables the expansions of these hypergeometric functions are finite polynomials.

4.1 Divergent part of effective potential

In this section we find the divergent contributions to the chiral effective potential using two approaches. The first approach based on direct calculation of expression (3.3) while the second approach uses eq. (3.57). Of course, both approaches give the coincident results.

There are at least two possibilities for calculation of the divergencies. The first one is based on direct expansion of the logarithm in (3.3) into power series of the matrix, i.e. $\ln(1+X) = X - X^2/2 + X^3/3 - X^4/4 + \cdots$, and calculating the traces for corresponding matrix powers. For the simplicity we rewrite matrix $X = H_0^{-1}H_1$ from (3.3) as a block type matrix

$$X = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \qquad \begin{array}{ll} A = \frac{-\bar{m}}{\Box_+}\frac{D^2\bar{D}^2}{\Box}(g\Phi + MQ^2)\,, & B = \frac{D^2}{\Box_+}\bar{g}\bar{\Phi}\,, \\ D = \frac{-m}{\Box_+}\frac{\bar{D}^2D^2}{\Box}\bar{g}\bar{\Phi}\,, & C = \frac{\bar{D}^2}{\Box_+}(g\Phi + MQ^2)\,. \end{array} \tag{4.11}$$

The trace of the first power of the matrix doesn't contain the field dependence. One can see that divergent terms $\sim M$ will appear only in 2, 3 and 4-th orders. We will consider them one after another.

In the second order one gets $-\frac{1}{2}\cdot 2MD^2\bar{D}^2(\bar{m}^2 g\Phi\frac{1}{\Box\Box_+^2} + \bar{g}\bar{\Phi}\frac{1}{\Box_+^2})Q^2$

$$\frac{\mathrm{Tr}(X^2)}{2} \sim -\frac{1}{2(4\pi)^2}M(\bar{m}^2 g\Phi + 2\bar{g}\bar{\Phi}m\bar{m})\ln\left(\frac{m\bar{m}}{L^2}\right). \tag{4.12}$$

The third order gives $\frac{-\bar{m}}{\Box_+^3}D^2\bar{D}^2\bar{g}\bar{\Phi}2g\Phi MQ^2 + \frac{D^2\bar{D}^2}{\Box_+^3}(-m\bar{g}^2\bar{\Phi}^2MQ^2)$

$$\frac{\mathrm{Tr}(X^3)}{3} \sim -\frac{1}{2(4\pi)^2}(\bar{m}2\bar{g}g\Phi\bar{\Phi} + m\bar{g}^2\bar{\Phi}^2)M\ln\left(\frac{m\bar{m}}{L^2}\right) \tag{4.13}$$

The fourth order gives $-D^2\bar{D}^2\frac{\Box}{\Box_+^4}\bar{g}^2\bar{\Phi}^2 g\Phi MQ^2$

$$\frac{\mathrm{Tr}(X^4)}{4} = -\frac{1}{2(4\pi)^2}g\bar{g}^2\Phi\bar{\Phi}^2M\ln\left(\frac{m\bar{m}}{L^2}\right) \tag{4.14}$$

Here L is a regularization scale. Combining these terms together one obtains the following expression

$$W_{\mathrm{div}} = -\frac{1}{2(4\pi)^2}\{\bar{m}^2 g\Phi + 2m\bar{g}\bar{G} + 2g\bar{g}\Phi\bar{G}\}M\ln\left(\frac{m\bar{m}}{L^2}\right), \tag{4.15}$$

where $\bar{G} = -D^2\Phi = Q^2\Phi$ is the earlier introduced combination, which was used in the equations of motion (2.3). The result (4.15) corresponds to the one given in [17] up to regularization scheme.

The second approach for calculation of divergencies implies use of the expansion (3.57). To get the divergencies it is enough to consider in the sum of (4.2) only terms coming from R_0:

$$R_0^{\rm div} = \int d^4x d^2\theta \frac{m}{2(4\pi)^2}(\bar{m}^2 + 2\bar{g}Q^2\Phi)\left(\frac{1}{\lambda} + hQ^2\Phi\right)\ln\left(\frac{m\bar{\mu}}{L^2}\right), \tag{4.16}$$

and R_1:

$$R_1^{\rm div} = \int d^4x d^2\theta \frac{g}{2(4\pi)^2}(\bar{m}^2 + 2\bar{g}Q^2\Phi)\left(\frac{1}{\lambda} + hQ^2\Phi\right)\Phi\ln\left(\frac{m\bar{\mu}}{L^2}\right). \tag{4.17}$$

In the above expressions we used relations $(\bar{m}+\bar{g}\bar{A})^2 = \bar{m}^2 + 2\bar{g}\bar{G}$ and $\bar{G} = Q^2\Phi$. The sum of R_0 and R_1 gives the result which coincides with (4.15) up to finite terms and divergent terms independent of Φ. We point out also that the expression W_{div} (4.15) is completely consistent with all earlier results on one-loop divergencies in the model under consideration obtained by the other methods [17].

4.2 Structure of finite contributions to chiral effective potential

In this section we demonstrate using (3.57) the explicit calculations of several finite contributions to effective potential with higher orders of external grassmannian momenta. According to the previous discussion, these terms can be presented as elements of a ring structure $R(\Phi, D^2\Phi, D\Phi D\Phi)$ (see also ref. [34]). From (4.8), (4.9), (4.10) one can see that the terms R_n in the expansion (4.2) should have the general form

$$R_n = \int d^4x d^2\theta \sum_{k=0}^{n} C_{k;n}(M, \bar{\mu}, m)\Phi^{n-k}\left(\frac{M}{m}D^2\Phi\right)^k, \qquad \Phi = \Phi(y,\theta) \tag{4.18}$$

here as before $M = hQ^2\Phi + 1/\lambda$, $\bar{\mu} = \bar{m} + \bar{g}\bar{\Phi}$, and $C_{k;n}(M, \bar{\mu}, m)$ are some functions. It can be also mentioned from (3.57) that $\Gamma_{(1)}$ is proportional to $M = hQ^2\Phi + 1/\lambda$ and at $h = 0$ and $1/\lambda = 0$ we get $\Gamma_{(1)} = 0$, i.e. then $\mathcal{N} = 1$ supersymmetry is recovered, the chiral effective potential is absent in agreement with the nonrenormalization theorem.

We will show that calculations of the finite contributions R_n to the chiral effective potential is closely related with so called Mellin-Barnes representation of hypergeometrical functions of several variables [35]. The hypergeometrical functions of grassmannian variables are the finite order polynomials which can be always expressed via a star-product. It means that the effective potential can be organized as a sum of elements of the mentioned ring.

4.2.1 R_2 contribution to the effective potential

For the beginning one notes that all terms R_n for $n > 1$ in (3.57) are finite. We will consecutively consider several first finite contributions to the chiral effective potential. In this section we find an explicit expression for R_2 term from the expansion (4.2) which defined by $E_{(2)}$ contribution in eq. (3.57). Since the expansion of $E_{(2)}$ contains the momenta variables, after the inverse Fourier transform the result will contain terms with derivatives, i.e. $\pi_i\Phi_i \to D\Phi$. Of course the result contains a constant contribution which is summed in (4.31), but it also contains the contributions with derivatives of the background fields.

A method of R_n terms calculation was described in the beginning of section 4. It supposes using the $E_{(n)}$ expansion in the form (4.9). However one can start directly with the exponential form (4.7).

In the expansion (4.2) we consider R_2 contribution containing one variable $l_{12} = \pi_2$. According to (3.57) and (4.8) it will be defined by $E_{(2)}$, which is the function of one variable l_{12}, as it demonstrated in (4.9). If we consider the Mellin transform (see e.g. [35]) for functions dependent on one variable

$$f(\rho) = \int_0^\infty f(l_i^2)(l_i^2)^{\rho-1} dl_i^2\,, \qquad f(l_i^2) = \int_{C_-} \frac{d\rho}{2\pi i} f(\rho)(l_i^2)^{-\rho}\,, \tag{4.19}$$

we can note that $R_{(2)}$ is the Mellin transform of function $E_{(2)}$, with $l_i = l_{12}$, $f(\rho) = \frac{\Gamma(\rho)}{(-TM\alpha_1\alpha_2)^\rho}$. The integration contour C_- in the complex ρ-plane goes from $-\infty$ and must separate "right" set of poles in the integrand from "left" set of poles. Further we will understand all contour integrals in this sense.

After integration over the proper-time T and α_1, α_2 one obtains the expression:

$$R_2 = \frac{g^2}{4(4\pi)^2} \cdot \frac{\bar{\mu}^2 M}{m} \int d^4x \int d\pi_1 d\pi_2\, \Phi(y,\pi_1)\Phi(y,\pi_2)\delta(\pi_1+\pi_2) I_2\,, \tag{4.20}$$

where

$$I_2 = \left(\frac{d}{ds}\right)\Big| \int_C \frac{d\rho}{2\pi i} \left(-l_{12}^2 \frac{M}{m}\right)^{-\rho} \frac{\Gamma(s+1-\rho)}{\Gamma(s)} \frac{\Gamma(\rho)\Gamma^2(1-\rho)}{\Gamma(-2\rho+2)}\,. \tag{4.21}$$

Here $l_{12} = \pi_1$ (see denotation (4.5)) and $\Gamma(\rho)$ the Euler gamma function [35]. Implementing $d/ds|$ and using the known properties of $\Gamma(2z)$ one obtains

$$I_2 = \int \frac{d\rho}{2\pi i} \left(-l_{12}^2 \frac{M}{m}\right)^{-\rho} \frac{\sqrt{\pi}}{2} \frac{\Gamma^2(1-\rho)\Gamma(\rho)}{\Gamma(\frac{3}{2}-\rho)}\,,$$

which is a representation for the hypergeometric function[5] ${}_2F_1(1,1;3/2;z)$ (see e.g. [35]). Finally we can rewrite this contribution as follows

$$\begin{aligned} R_2 &= \frac{g^2}{4(4\pi)^2} \cdot \int d^4x d^2\theta \frac{M}{m} \bar{\mu}^2 \Phi(y,\theta) {}_2F_1\left(1,1;\frac{3}{2};\frac{M}{4m}D^2\right)\Phi(y,\theta) \\ &= \frac{g^2}{4(4\pi)^2} \int d^4x d^2\theta\, \frac{M}{m}\bar{\mu}^2 \left(\Phi^2 + \frac{M}{6m}\Phi D^2\Phi\right), \end{aligned} \tag{4.22}$$

we remind that $\bar{\mu} = \bar{m} + \bar{g}\bar{\Phi}$, $M = h(Q^2\Phi) + 1/\lambda$ and use the explicit expression for ${}_2F_1(1,1;\frac{3}{2};z) = \frac{\arcsin\sqrt{z}}{\sqrt{z(1-z)}} = 1 + \frac{1}{3!}\frac{M}{m}l^2$ where $z = l_{12}^2 \frac{M}{m}$ in the given case.

In bosonic sector this contribution has the simple form

$$R_2^b = \frac{g^2}{4(4\pi)^2} \int d^4x\, \frac{M}{m}\bar{m}^2 \left(2AF + \frac{M}{6m}F^2\right), \qquad M = hF + \frac{1}{\lambda}\,. \tag{4.23}$$

[5] To see this, one should change $\rho \to -\rho$ and $C_- \to C_+$.

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4.2.2 R_3 contribution to the effective potential

It is remarkable that all terms $R_{(n)}$ in the expansion (4.2) can be founded using the method demonstrated in the previous section (i.e. using the Mellin transform). For higher R_n the only difference concerns only number of variables.

In the present section we propose a general method for calculation of R_n contributions based on the mentioned Mellin-Barnes representation for hypergeometric functions of several variables [35]. In order to illustrate the main idea, we consider contribution to the effective potential from R_3, which contains three variables. Substituting the expression E_3 given in (4.9) to general expansion (3.57) one obtains

$$\begin{aligned}
R_3 &= \frac{g^3 M}{6(4\pi)^2}\cdot\frac{\bar{\mu}^2}{m^2}\int d^4x\int\prod d^2\pi\,\Phi(y,\pi_1)\Phi(y,\pi_2)\Phi(y,\pi_3)\delta\left(\sum_i\pi_i\right)I_3\,,\\
I_3 &= \frac{\sqrt{\pi}}{4}\int\frac{d^3\rho}{(2\pi i)^3}\left(-l_{12}^2\frac{M}{4m}\right)^{\rho_1}\left(-l_{13}^2\frac{M}{4m}\right)^{\rho_2}\left(-l_{23}^2\frac{M}{4m}\right)^{\rho_3}\times\\
&\quad\times\Gamma(-\rho_1)\Gamma(-\rho_2)\Gamma(-\rho_3)\frac{\Gamma(\rho_1+\rho_2+1)\Gamma(\rho_1+\rho_3+1)\Gamma(\rho_2+\rho_3+1)}{\Gamma(\rho_1+\rho_2+\rho_3+\frac{3}{2})}\,,
\end{aligned}\tag{4.24}$$

where l_{ik} are defined by (4.5): $l_{12}^2=\pi_2^2=-\frac{1}{2}\pi_2(\pi_1+\pi_3)$, $l_{13}^2=\pi_1^2=-\frac{1}{2}\pi_1(\pi_2+\pi_3)$, $l_{23}^2=\pi_3^2=-\frac{1}{2}\pi_3(\pi_1+\pi_2)$, and I_3 is the Mellin-Barnes representation for hypergeometric functions of several variables (three variables in the case under consideration) [35]. It should be noted that variables l_{12}, l_{23}, l_{13} appear in the expression (4.24) symmetrically. Using the definition for the hypergeometric function ${}_2F_1$ of two variables [35] one can write

$$\begin{aligned}
I_3 &= \sum_{n_1,n_2=0}^{\infty}\frac{\sqrt{\pi}}{4}\left(l_{12}^2\frac{M}{4m}\right)^{n_1}\left(l_{13}^2\frac{M}{4m}\right)^{n_2}\frac{\Gamma(n_1+n_2+1)}{\Gamma(n_1+n_2+\frac{3}{2})}\times\\
&\quad\times{}_2F_1\left(n_1+1,n_2+1;n_1+n_2+\frac{3}{2};l_{23}^2\frac{M}{4m}\right).
\end{aligned}\tag{4.25}$$

Properties of hypergeometric functions allows us to rewrite the last expression as follows

$$I_3=\frac{\sqrt{\pi}}{4}\sum_{n=0}^{\infty}(u)^n\frac{\Gamma(n+1)}{\Gamma(n+\frac{3}{2})}F_1\left(1,n+1,n+1;n+\frac{3}{2};s,t\right),\tag{4.26}$$

where

$$u=l_{12}^2\frac{M}{4m}\,,\qquad s=l_{23}^2\frac{M}{4m}\,,\qquad t=l_{13}^2\frac{M}{4m}\,.$$

The definitions for l_{kn} are given in (4.5). Further we use F_1-type function or the Appel's hypergeometrical function from, so called, the Horn list of functions [35].

Using the property $(l_{i_1i_2}^2)^k=0$, $k>1$, one can rewrite eq. (4.26) as a finite order polynomial. It leads for the 3-legs contribution to

$$\begin{aligned}
R_3 &= \frac{-g^3}{6(4\pi)^2}\left(\frac{M}{m^2}\right)\bar{\mu}^2\int d^4x\int\prod d^2\pi\,\Phi(y,\pi_1)\Phi(y,\pi_2)\Phi(y,\pi_3)\delta\left(\sum_i\pi_i\right)\times\\
&\quad\times\left(\frac{1}{2}+\frac{1}{3\cdot4}(s+t+u)+\frac{1}{3\cdot4\cdot5}(st+su+tu)+\frac{1}{5\cdot6\cdot7}stu\right).
\end{aligned}\tag{4.27}$$

Taking into account the definitions for u, t, s, the definitions (4.6), the expansion (4.8) and discussion concerning the inverse Fourier transform given after eq. (4.10), we transform the expression for R_3 in the coordinate representation

$$R_3 = \frac{g^3}{6(4\pi)^2}\int d^4x d^2\theta \left(\frac{M}{m^2}\right)\bar{\mu}^2 \times \tag{4.28}$$
$$\times\left(\frac{1}{2}\Phi^3+\frac{1}{4}\frac{M}{4m}\Phi^2 D^2\Phi+\frac{1}{4\cdot 5}\left(\frac{M}{4m}\right)^2\Phi D^2\Phi D^2\Phi+\frac{1}{5\cdot 6\cdot 7}\left(\frac{M}{4m}\right)^3(D^2\Phi)^3\right),$$

where as before $\bar{\mu}=\bar{m}+\bar{g}\bar{\Phi}$, $M=hQ^2\Phi+1/\lambda$.

In the bosonic sector this result looks like

$$R_3^b = \frac{g^3}{6(4\pi)^2}\int d^4x \,\frac{M\bar{m}^2}{m^2}\left(\frac{3}{2}A^2F+\frac{1}{2}\frac{M}{4m}AF^2+\frac{1}{4\cdot 5}\left(\frac{M}{4m}\right)^2F^3\right),\qquad M=hF+\frac{1}{\lambda}. \tag{4.29}$$

It is easy to understand that exploiting the Mellin transform for higher R_n contributions, we will obtain the expressions containing the hypergeometric function of several variables. Such functions for $n>2$ are called generalized Lauricella hypergeometric functions [35, 36]. This is the expected result. The general expressions for all one-loop massive Feynman diagrams have been studied in refs. [36] using the Mellin-Barnes representation for hypergeometric functions of several variables by contour integrals and it was shown that all of them can be expressed via the Lauricella hypergeometric functions and the described procedure, in principle, can be applied for calculations of any higher R_n contributions to chiral effective potential.

4.3 Constant field contribution to the chiral effective potential

In the previous sections we considered two examples which demonstrated that, in principal, all R_n can be calculated and the results can be expressed via hypergeometric functions of several variables. Now we construct an approximation taking into account some terms from all R_n Since every R_n is a finite order polynomial in powers of $D^2\Phi$ (see the discussions in the beginning of section (4.2) we can try, as a first approximation, to sum up all terms without these derivatives and obtain an approximate expression for the chiral effective potential containing the contributions from all R_n.

The expansion (4.8)[6] always begins from constant, i.e. $E_{(n)}=1+\cdots$. Let's choose from all $E_{(n)}$ only the terms without variables l_{ij} and calculate their contribution to (3.57). It will correspond to the situation then we take into account only $k=0$ term in the representation (4.18) for all R_n, i.e. we sum contributions from all R_n which have no grassmannian derivatives.

One can see that the sum of contributions to the chiral effective potential (3.57) containing no Grassmann derivatives is written in the following form

$$\Gamma^{(0)}_{(1)} = \frac{1}{2(4\pi)^2}\int d^4x d^2\theta\, m\bar{\mu}^2 M\sum_{n=2}^{\infty}\left(-\frac{g\Phi(y,\theta)}{m}\right)^n\frac{1}{n(n-1)}, \tag{4.30}$$

where multiplier $\frac{1}{n(n-1)}$ appears as the result of the proper time integration in (3.57).

[6] See also examples (4.9).

The sum can be calculated exactly, that gives

$$\Gamma^{(0)}_{(1)} = \frac{1}{2(4\pi)^2}\int d^4x d^2\theta\, m\left(\bar{m}+\bar{g}\bar{\Phi}\right)^2\left(hQ^2\Phi+\frac{1}{\lambda}\right)\left(-\frac{g\Phi}{m}+\left(1+\frac{g\Phi}{m}\right)\ln\left(1+\frac{g\Phi}{m}\right)\right). \tag{4.31}$$

The expression (4.31) is the chiral effective potential in approximation when all terms containing the $D^2\Phi$ can be neglected, however all terms without these derivatives are exactly summed up. The corrections to this approximation obligatory contain the terms with Grassmann derivatives of the background field.

Now we study a structure of the effective potential (4.31) in the bosonic component sector. We have

$$\Gamma^{(0)}_{(1)}|_b = \frac{m\bar{m}^2}{2(4\pi)^2}\int d^4x\left(hF+\frac{1}{\lambda}\right)\frac{g}{m}F\ln\left(1+\frac{g}{m}A\right). \tag{4.32}$$

This expression gives a correction to the classical potential (2.5)

$$\Delta V = \frac{m\bar{m}^2}{2(4\pi)^2}\left(hF+\frac{1}{\lambda}\right)\frac{g}{m}F\ln\left(1+\frac{g}{m}A\right). \tag{4.33}$$

Eliminating the auxiliary field from the classical equations of motions (2.4) one finds the full one-loop effective potential in this approximation

$$V^{(1)} = V+\Delta V = \bar{G}\left(G+\frac{1}{\lambda}\bar{G}+\frac{h}{6}\bar{G}^2\right)+\frac{g\bar{m}^2}{2(4\pi)^2}\left(-2h\bar{G}+\frac{1}{\lambda}\right)\ln\left(1+\frac{2g}{m^2}G\right), \tag{4.34}$$

where $G = mA+\frac{g}{2}A^2$, $\bar{G} = \bar{m}\bar{A}+\frac{\bar{g}}{2}\bar{A}^2$. The exact one-loop effective potential includes, besides (4.31), the powers of $D^2\Phi$. It means, the exact component form of the effective potential will contain, besides (4.33) the powers of F. Therefore, after eliminating the $F,\bar{F}$-terms from classical equations of motion, one gets the powers of $G,\bar{G}$ as correction terms in (4.34).

5. Summary

We have developed the general approach for finding the one-loop effective potential in $\mathcal{N} = 1/2$ noncommutative Wess-Zumino model. Using the symbol-operator techniques we obtained a general expression for the effective potential in terms of a superfield heat kernel. This heat kernel was exactly calculated for the specific background superfields determining the functional dependence of the chiral effective potential. As a result we found the exact form of the effective potential including the complete dependence on Φ and $D^2\Phi$ in term of a single proper time integral. This effective potential contains all previously obtained results for the theory under consideration as partial cases.

To clarify the structure of the effective potential in more details we have constructed a systematic procedure of expansion of the effective potential in a power series over Φ and $D^2\Phi$. Each term of this expansion can be calculated in an explicit form. We demonstrated the calculations of divergent contributions to the one-loop effective potential as well as a few first finite contributions. All finite contributions to the effective potential are expressed in terms of hypergeometrical functions of several variables.

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The expansion of the effective potential has an enough simple structure and allows to organize resummation of the above series and to get a series in derivatives $D^2\Phi$ with the coefficients depending on Φ. We have demonstrated how to obtain the first term in this new expansion containing no derivatives but including all powers of Φ.

To conclude, we carry out the complete analysis of the one-loop effective potential in $\mathcal{N} = 1/2$ WZ model. The final exact solution for this effective potential is constructed and various its approximate forms are found.

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